Rasmussen Set Theory (RST)

RST = ZF + Existence - (Separation + Replacement)

Existence Schema:

$\forall w_1 \ldots w_n \forall x \exists s\ (x \in s \equiv (\varphi(w_1 \ldots w_n, x, s))) \vee (x \in s \equiv (\varphi(w_1 \ldots w_n, x, s)))$, where 'φ' is a formula in the language of set theory with free variables among x, w_1, ... , w_n, s.

In other words, for any formula ψ in the language of set theory, there is a set whose elements either all satisfy the formula or all do not satisfy the formula.

A formula is 'natural' if there is a set all of whose elements satisfy it.

Extensionality:
$\forall x \forall y (\forall z (z \in x \equiv z \in y) \rightarrow x = y)$

This axiom asserts that when sets *x* and *y* have the same members, they are the same set.

Pairs:
$\forall x \forall y \exists z \forall w (w \in z \equiv w = x \vee w = y)$

This axiom asserts that if given any set *x* and *y*, there exists a pair set of *x* and *y*, i.e., a set which has only *x* and *y* as members:

Unions:
$\forall x \exists y \forall z (z \in y \equiv \exists w (w \in x \ \&\ z \in w))$

The next axiom asserts that for any given set *x*, there is a set *y* which has as members all of the members of all of the members of *x*:

Power Set:
$\forall x \exists y \forall z [z \in y \equiv \forall w (w \in z \rightarrow w \in x)]$

This axiom asserts that for any set *x*, there is a set *y* which contains as members all and only those sets whose members are also elements of *x*, i.e., *y* contains all of the subsets of *x*:

Regularity:
$\forall x (\exists y (y \in x \ \&\ \forall z (z \in x \rightarrow \neg (z \in y))))$

A member *y* of a set *x* with this property is called a 'minimal' element. This axiom rules out the existence of circular chains of sets (e.g., such as $x \in y$ & $y \in z$ & and $z \in x$) as well as infinitely descending chains of sets (such as $\ldots x_3 \in x_2 \in x_1 \in x_0$).