

A Paradox for Platonic Propositions

Abstract

I give a new version of Russell's paradox concerning *propositions*. My formulation consists of an argument that purports to deduce a contradiction from the premise that for any xs , there is the proposition that those xs exist. The argument poses a special problem for the usual *Platonist* views about propositions, since Platonists standardly take there to be a proposition for every fact about the world, including facts about what exists. The upshot is that we need a better theory of abstract propositions.

Key Words

Russell's Paradox, Abstract Propositions, Propositions

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1. Introduction

Bertrand Russell introduced a little discussed paradox concerning propositions at the end of his *Principles of Mathematics*. The paradox concerns propositions that aren't about themselves (more precisely: propositions that aren't members of the class of things they quantify over). He argues, basically, that the proposition that is about exactly those propositions that aren't about themselves is *itself* about itself *iff* it isn't; hence, we have a contradiction. Russell confesses not to have a solution—not even in terms of his theory of types. He writes, “What the complete solution of the difficulty may be, I have not succeeded in discovering; but as it affects the very foundations of reasoning, I earnestly commend the study of it to the attention of all students of logic.”¹

Although philosophers and logicians have paid much attention to Russell's related set-theoretic paradox, surprisingly little attention has been given to his propositions paradox. There are, to be sure, discussions of the logic of the propositions paradox.² But articles on the philosophical import are scant.³

In this article, I will give a new formulation of Russell's paradox. I will show that my formulation poses a special problem for *Platonist* theories of propositions.

2. The Aboutness Paradox

I begin by stipulating the following definition: ' x is a proposition about itself' =_{def} 'there are some ys , such that x is the proposition that the ys exist, and x is one of the ys '. With this definition in hand, consider the following argument, which I call 'the Aboutness Paradox':

1. For all xs , there is the proposition that the xs exist.
2. There are some ys , such that the ys are all and only the propositions that are not about themselves.
3. Therefore, there is the proposition P that the ys exist. (1, 2)
4. If P is a proposition about itself, then P is one of the ys —and thus, P is *not* a proposition about itself (by definition).
5. If P is not a proposition about itself, then P is not one of the ys —and thus P is a proposition about itself (by definition).
6. Therefore, there is a proposition that is about itself *if and only if* it is not. (3–5)

¹ Russell 1913: 527.

² Examples include Cocchiarella 2000, Klement 2001, Cantini 2004.

³ Cf. Plantinga & Grim 1993.

This argument retains the spirit of Russell's argument concerning propositions. But there are a couple notable differences. First, unlike Russell's argument, the Aboutness Paradox makes no reference to *classes*. It is instead expressed using plural quantification. Second, the Aboutness Paradox has fewer premises; in particular, it makes no use of a certain identity condition that is built into Russell's argument.⁴

Let us have a closer look at the premises. Start with premise 1: for any *xs*, there is the proposition that the *xs* exist. This premise seems to fall right out of the Platonist theory that propositions are necessarily existing abstracta. For suppose propositions are necessary entities. Then it is plausible that for *any* fact, there is a proposition that corresponds to (or *is*) that fact. If that is right, then it is plausible that for any fact of the form *that the xs exist*, there is a proposition of that same form. Premise 1 follows. (I will consider a different, "nominalist" version of the premise in the next section.)

Turn, next, to premise 2: there are some *ys*, such that the *ys* are all and only the propositions that are not about themselves. Here is an argument for premise 2:

- 2.1. $\langle 2 + 1 = 3 \rangle$ is a proposition.
- 2.2. $\langle 2 + 1 = 3 \rangle$ is not about itself (by the stipulated definition).
- 2.3. Therefore, there is a proposition that is not about itself. (2.1, 2.2)
- 2.4. Therefore, there are *the* propositions that are not about themselves.
- 2.5. Therefore, there are some *ys*, such that the *ys* are all and only the propositions that are not about themselves.

I take each step in this argument to be plausible (inescapable, really). Someone might reject the inference from 2.4 to 2.5 on the grounds that they can't make sense of plural quantification. But my impression is that most philosophers find plural quantification unproblematic. Moreover, if it turns out that plural quantification is the weakest link in the argument, then that will certainly be surprising and noteworthy news in its own right.

Let us move on to premise (4): if *P* is a proposition about itself, then *P* is one of the *ys*—and thus, *P* is *not* a proposition about itself. This premise packs two premises together:

- 4.1. If *P* is a proposition about itself, then *P* is one of the *ys*.
- 4.2. If *P* is one of the *ys*, then *P* is not about itself.

To see that premise (4.1) is true, first recall that '*P*' designates the proposition that the *ys* exist. Next, recall the stipulated definition of 'about': '*x* is a proposition about itself' =_{def} 'there are some *ys*, such that *x* is the proposition that the *ys* exist, and *x* is one of the *ys*'. From these definitions it follows that *P* can only be about itself if *P* is one of the *ys* it says exist. Therefore, if *P* is about itself, then *P* is one of those *ys*. Premise (4.2) is also true: the *ys* are (by definition)

⁴ The identity condition Russell uses is this: if class *n* is different from class *m*, then \langle every member of *n* is true \rangle is not the same proposition as \langle every member of every *m* is true \rangle .

precisely those propositions that are not about themselves. So if P is one of the ys , then P is not about itself. These premises are analytic.

The final premise is (5): if P is not a proposition about itself, then P is not one of the ys —and thus P is a proposition about itself. This premise, like the previous one, follows from the definitions of terms. To see this, suppose P is not a proposition about itself. Then P can't be one of the ys ; if it were, then, by the definition of 'about', P would be about itself. So, P is not one of the ys . But then since the ys are just those propositions that are not about themselves, it follows that P is not a proposition that is not about itself. This result together with (4) implies the absurd conclusion: P both is and is not about itself.

In my estimation, the Aboutness Paradox poses a significant and unforeseen problem for Platonist theories of propositions. Premise 1 is hard to resist if propositions are necessarily existing things, as Platonists think. Premise 2 is appealing on any view of propositions. And the remaining premises follow from the definitions of terms. It seems to me, therefore, that we will not solve the Aboutness Paradox without pursuing a deeper understanding of the nature of propositions.⁵

3. A Way of Escape

If you are a nominalist, you have an easy way to escape the argument: simply deny the first premise.

But we should consider whether there might be an argument in the neighborhood that poses a problem even for nominalist theories of propositions. Here is my best attempt to convert the Aboutness Paradox into an argument that targets nominalists:

7. For any xs , possibly, there is the proposition that the xs exist.
8. There are some ys , such that the ys are all and only the propositions that are not about themselves.
9. Therefore, possibly, there is the proposition P that the ys exist. (1, 2)
10. Necessarily, if P is a proposition about itself, then P is one of the ys —and thus, P is *not* a proposition about itself (by definition).
11. Necessarily, if P is not a proposition about itself, then P is not one of the ys —and thus P is a proposition about itself (by definition).
12. Therefore, possibly, there is a proposition that is about itself *if and only if* it is not. (3–5)

⁵ This argument concerning propositions is, in my opinion, more challenging than Russell's paradox concerning sets. Concerning sets, we may carefully craft axioms of set-hood (such as Zermelo-Fraenkel's axioms) that preclude the existence of sets that are members of themselves. But it is not so clear how crafting axioms concerning aboutness could help us see where *this* argument goes wrong. After all, the argument makes no explicit assumptions about the nature of aboutness: although the argument is given in terms of 'about', I could have given the argument in terms of 'about*', using the same stipulated definition. Thus, it seems the puzzle concerning propositions is importantly different from the puzzle concerning sets.

This argument is in a certain respect much weaker than the previous one: it merely requires that for any given *xs*, there *can* be a proposition about those *xs*. Even nominalists could find that plausible.

The argument flounders, however, at premise (10). The problem is with the necessity operator, which is needed to ensure validity. Premise (10) entails that necessarily, if *P* is one of the *ys*, then *P* is *not* a proposition about itself. But that isn't true, for the *ys* don't *necessarily* comprise all and only the propositions that aren't about themselves. The *ys* merely comprise the propositions that aren't about themselves *in our world*. Possibly, those same propositions exist in another world in which there are *additional* propositions that aren't about themselves, such as *P*. Then *P* could fail to be about itself without thereby being one of the *ys*. The paradox is thus blocked here. (The original Aboutness Paradox doesn't face this problem because it has no use for the necessity operator.)

I have not found other promising arguments in the neighborhood that might appeal to nominalists. It appears, therefore, that nominalists may escape the Aboutness Paradox and its neighbors unscathed.

Platonists, on the other hand, are in a different boat. It is not easy to see how they may resist the premises. The weakest link is premise (1): that for any *xs*, there is the proposition that the *xs* exist. (The other premises are analytic, assuming the intelligibility of plural quantification.) Yet there is a plausible argument for (1):

- 1.1. For all *xs*, the *xs* exist.
- 1.2. For all *xs*, if the *xs* exist, then it is true that the *xs* exist.
- 1.3. For all *xs*, if it is true that the *xs* exist, then there is the proposition that the *xs* exist.
- 1.4. Therefore, for all *xs*, there is the proposition that the *xs* exist.

Premise (1.1) is trivially true if we suppose (stipulate) that the extension of 'existence' is the domain of quantification. Premise (1.2) follows from an instance of the T-schema: '*p*' is true *iff* *p*. Platonists generally accept this. And Premise (1.3) follows from the Platonist conviction that wherever there is *truth* there is a true proposition.

The best hope for escaping the argument, it seems to me, is to motivate a reason to deny (1.2) or (1.3). One way to do that is to suppose that *truths* about reality depend upon *thoughts*. Perhaps thoughts are the primary bearers of truth. In that case, existence wouldn't automatically entail truth: truth would require a further act of thinking. Then, since no one actually has a thought that is about each and every particular thought, premise (1) is false.⁶

Platonists, however, who take propositions to be mind-independent abstracta, will have to find another way out. Referring

⁶ One might think that even an omniscient mind, which would know all true thoughts, couldn't have a *thought* about each and every particular thought it knows.

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